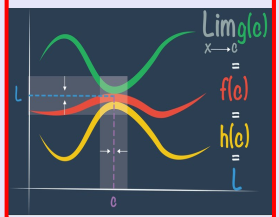
Calculus I

Lecture 41



Feb 19-8:47 AM

$f(x) = x^5 - 5x$

Polynomial Function $\to$ Cont. & Diff. $(-\infty, \infty)$

$\to$ Domain $(-\infty, \infty)$

$f(-x) = (-x)^5 - 5(-x) = -x^5 + 5x = -(x^5 - 5x) = -f(x)$

Since $f(-x) = -f(x)$ $\to$ odd Function

$\to$ Symmetric w/t origin

Y-Int $\to x = 0 \to y = f(0) = 0 \Rightarrow (0,0)$

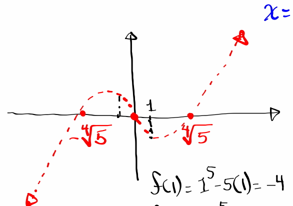
x-Int $\to y = 0 \to f(x) = 0 \to x^5 - 5x = 0$

$x(x^4 - 5) = 0$

$x = 0 \leftarrow$ $\quad$ $\hookrightarrow x^4 - 5 = 0$

$x^4 = 5$

$x = \pm\sqrt[4]{5}$



$f(1) = 1^5 - 5(1) = -4$

$f(-1) = (-1)^5 - 5(-1) = 4$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

Apr 29-8:46 AM

Class QZ 19

$f(x) = x^5 - 5x$

1) Find $f'(x)$, Solve $f'(x)=0$

$f'(x) = 5x^4 - 5$
$= 5(x^4 - 1)$
$= 5(x^2+1)(x+1)(x-1)$

$\boxed{f'(x)=0 \rightarrow x = \pm 1}$

2) Find $f''(x)$, Solve $f''(x)=0$

$f''(x) = 20x^3$

$\boxed{f''(x)=0 \rightarrow x=0}$

3) Complete the Sign chart.

x	$-\infty$		-1		0		1		∞
$f'(x)$		+	•	−		−	•	+	
$f''(x)$		−		−	•	+		+	
$f(x)$		↗		↘		↘		↗	

$(-1,4)$ Max $\quad (0,0)$ I.P. $\quad (1,-4)$ Min

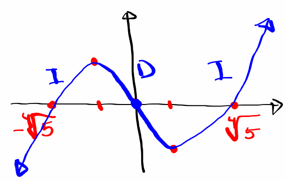
4) Discuss in interval notation

increasing $(-\infty,-1)\cup(1,\infty)$

Decreasing $(-1,1)$

Concave up $(0,\infty)$

Concave down $(-\infty,0)$



Apr 25-9:31 AM

$f(x) = x^4 - 2x^2 + 1$

Polynomial function $\rightarrow$ Domain $(-\infty,\infty)$

Cont. & Diff. $(-\infty,\infty)$

$f(-x) = (-x)^4 - 2(-x)^2 + 1 = x^4 - 2x^2 + 1 = f(x)$

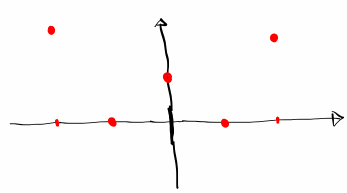
$f(x)$ is an even function $\rightarrow$ Symmetric w/t Y-axis.

Y-Int $\rightarrow x=0 \rightarrow f(0)=1 \rightarrow (0,1)$

x-Int $\rightarrow y=0 \rightarrow f(x)=0 \rightarrow x^4 - 2x^2 + 1 = 0$

$(x^2-1)(x^2-1)=0$

$x = \pm 1$



$f(2) = 2^4 - 2(2)^2 + 1$
$= 16 - 8 + 1 = 9$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

Apr 29-8:59 AM

$f(x) = x^4 - 2x^2 + 1$

$f'(x) = 4x^3 - 4x$

$f''(x) = 12x^2 - 4$

$f'(x) = 0$

$4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$\hookrightarrow x = 0 \qquad \hookrightarrow x = \pm 1$

C.P. $(0, f(0)) = (0, 1)$

$(1, f(1)) = (1, 0)$

$(-1, f(-1)) = (-1, 0)$

$f''(x) = 0$

$12x^2 - 4 = 0$

$4(3x^2 - 1) = 0$

P.I.P. $\hookrightarrow x = \pm \frac{\sqrt{3}}{3} \approx \pm .6$

x	$-\infty$ to -1	-1 to $-\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$ to 0	0 to $\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$ to 1	1 to ∞
$f'(x)$	−	+	+	−	−	+
$f''(x)$	+	+	−	−	+	+
$f(x)$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

Min ($x=-1$), I.P. ($x=-\frac{\sqrt{3}}{3}$), Max ($x=0$), I.P. ($x=\frac{\sqrt{3}}{3}$), Min ($x=1$)

C.P. $(0, 1), (1, 0), (-1, 0)$

I.P. $\left(-\frac{\sqrt{3}}{3}, f\left(-\frac{\sqrt{3}}{3}\right)\right), \left(\frac{\sqrt{3}}{3}, f\left(\frac{\sqrt{3}}{3}\right)\right)$

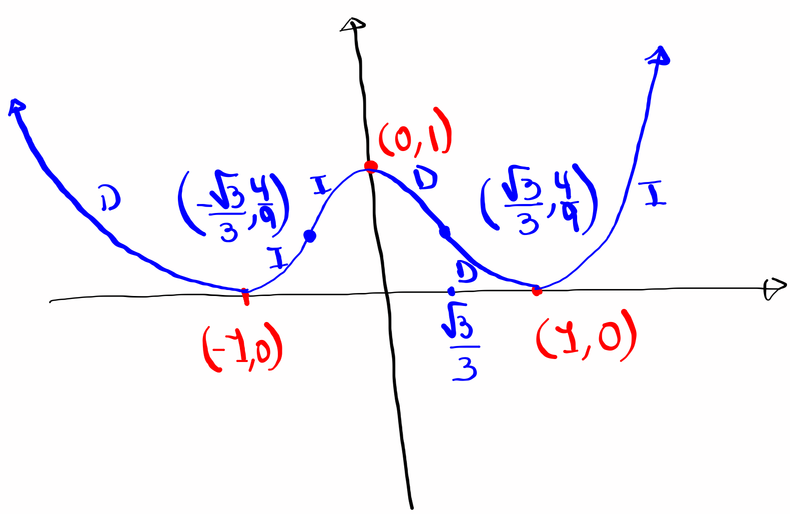
$f\left(\frac{\sqrt{3}}{3}\right) = f\left(\frac{1}{\sqrt{3}}\right) = \left(\frac{1}{\sqrt{3}}\right)^4 - 2\left(\frac{1}{\sqrt{3}}\right)^2 + 1$

$f(x) = x^4 - 2x^2 + 1 = (x^2 - 1)^2 = \left(\left(\frac{1}{\sqrt{3}}\right)^2 - 1\right)^2 = \left(\frac{1}{3} - 1\right)^2$

$= \left(\frac{-2}{3}\right)^2 = \frac{4}{9}$

I.P. $\left(-\frac{\sqrt{3}}{3}, \frac{4}{9}\right), \left(\frac{\sqrt{3}}{3}, \frac{4}{9}\right)$

Apr 29-9:05 AM



Apr 29-9:20 AM

$f(x) = \dfrac{x-2}{x+1}$ Domain All reals except $x+1=0$, $x=-1$

$(-\infty,-1)\cup(-1,\infty)$

Vertical Asymptote at $x=-1$

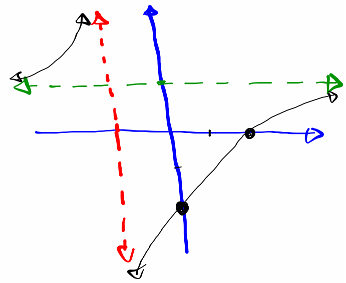
Y-Int $\to x=0 \to y=-2 \to (0,-2)$

x-Int $\to y=0 \to x-2=0 \to x=2 \to (2,0)$

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x-2}{x+1} = 1$

$\Rightarrow$ H.A. $\to y=1$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{x-2}{x+2} = 1$



Apr 29-9:23 AM

$f(x) = \dfrac{x-2}{x+1}$

$f'(x) = \dfrac{1(x+1)-(x-2)\cdot 1}{(x+1)^2} = \dfrac{3}{(x+1)^2}$

$f'(x) \neq 0$

$f'(x)$ is undefined at $x=-1$

$f'(x) > 0$

$f(x)$ is increasing.

$f'(x) = 3(x+1)^{-2}$

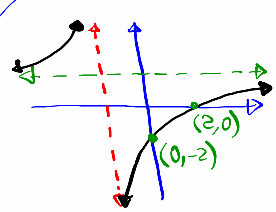
$f''(x) = 3\cdot(-2)(x+1)^{-3}\cdot 1$

$f''(x) = \dfrac{-6}{(x+1)^3}$

$f''(x) \neq 0$

$f''(x)$ is und. at $x=-1$

x	$-\infty$	-1	∞
$f'(x)$	+	o	+
$f''(x)$	+	o	−
$f(x)$			



Apr 29-9:28 AM

Find two positive numbers with the sum of 10 and the maximum product.

$x>0$ $\quad$ $y>0$

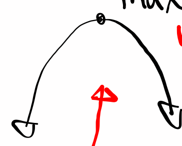
$x+y=10$

$y=10-x$

Max xy

Maximize $x(10-x)$

Max. Pt.



where $f'(x)=0$

$10-2x=0$

$\boxed{x=5}$

$\boxed{y=5}$

$f(x)=x(10-x)=10x-x^2$

$f'(x)=10-2x$

$f''(x)=-2<0 \rightarrow f(x)$ is C.D.

The two numbers are 5 & 5

Apr 29-9:40 AM

For $\varepsilon=.1$, find δ such that $\lim_{x\to 10} x^2=100$. ✓

$f(x)=x^2$

$a=10$

$L=100$

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|x^2-100|<\varepsilon$ " $|x-10|<\delta$

$|x+10|\,|x-10|<\varepsilon$ " $|x-10|<\delta$

$\underbrace{|x+10|}_{\text{Bound}}\ \underbrace{|x-10|}_{\text{Keep}}$

If $|x+10|<C$, then $|x-10|<\frac{\varepsilon}{C}$

If we wish to have $\delta\le 1$

$|x-10|<1$

$-1<x-10<1$

Add 20

$-1+20<x-10+20<20+1$

$19<x+10<21$

$|x+10|<21$

$C=21$

$\delta=\min\left\{1,\frac{\varepsilon}{21}\right\}$

For $\varepsilon=.1$

$\delta=\min\left\{1,\frac{.1}{21}\right\}$

$\boxed{\delta=\frac{1}{210}}$

Apr 29-9:45 AM